Written Exam at the Department of Economics winter 2016-17

Advanced Industrial Organization

Final Exam

22 december 2016

(3-hour closed book exam)

Brief solutions

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam consists of 4 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Please answer all 5 questions

1. *Durable good monopoly.* (a) Suppose in period 1, he sells q_1. In period 2, $1-q_1$ consumers have yet to buy. A consumer who did not buy in period 1 will buy in period 2 if $\theta \geq p_2$. Thus, in period 2, the demand is $q_2 = 1 - q_1 - p_2$. In period 1, a consumer would buy (rather than wait) if $\theta - p_1 + \frac{1}{2}\theta \geq -\frac{1}{2}p_2 + \frac{1}{2}\theta$, or $\theta \geq p_1 - \frac{1}{2}p_2$. Thus, $q_1 = 1 - p_1 + \frac{1}{2}p_2$, which means $q_2 = p_1 - \frac{3}{2}p_2$. Total profit is therefore

$$p_1 q_1 + \frac{1}{2}p_2 q_2 = p_1\left(1 - p_1 + \frac{1}{2}p_2\right) + \frac{1}{2}p_2\left(p_1 - \frac{3}{2}p_2\right)$$

Maximizing this expression with respect to p_1 and p_2 yields $p_1 = \frac{3}{4}$ and $p_2 = \frac{1}{2}$, so that $q_1 = \frac{1}{2}$ and $q_2 = 0$. (b) Suppose in period 1, the quantity sold is q_1. Then in period 2, the demand will be $q_2 = 1 - q_1 - p_2$. The period 2 profit is $\pi_2 = p_2(1 - q_1 - p_2)$. Maximizing this yields $p_2 = \frac{1-q_1}{2}$ so that $\pi_2 = \frac{(1-q_1)^2}{4}$. In period 1, the consumer buys (rather than waits) if $\theta \geq p_1 - \frac{1}{2}p_2$ so that $q_1 = 1 - p_1 + \frac{1}{2}p_2$. Therefore,

$$q_1 = 1 - \frac{4}{5}p_1$$

The firm's profit is therefore

$$p_1 q_1 + \frac{1}{2}\pi_2 = p_1\left(1 - \frac{4}{5}p_1\right) + \frac{4^2}{5^2}\frac{(p_1)^2}{8}$$

The first order condition yields $p_1 = \frac{25}{36}$ so that $q_1 = \frac{4}{9}$ and $q_2 = \frac{5}{18}$. Here he cannot achieve the commitment solution (which involves no sales in period 2) because in period 2, he would be tempted to cut the price and sell to some residual consumers. When consumers anticipate such price cuts in period 2, they are less willing to purchase in period 1, so he sells less in period 1 without commitment. .

2 *Non-linear pricing.* (a) If the monopolist sells 2 units to each consumer, $T(2)$ has to be at most \$50 to induce the low type to buy. So the profit would be \$50 per consumer. However, he can do better by screening the consumers, selling two units to the H type and one unit to the L type. The IC and participation constraints have to be satisfied:

$$80 - T(2) \geq 40 - T(1)$$

$$35 - T(1) \geq 50 - T(2)$$

$$80 - T(2) \geq 0$$

$$35 - T(1) \geq 0$$

As usual, the IC constraint for the high type, and the participation constraint for the low type, will bind:

$$80 - T(2) = 40 - T(1)$$

$$35 - T(1) = 0$$

Thus, Type L buys 1 unit and pays $T(1)$ =35; type H buys 2 units and pays $T(2) = 75$. The monopolist's expected profit per consumer is $\frac{1}{2}75 + \frac{1}{2}35 = 55$. (b) No. There is in fact a quantity premium because Type H pays 75/2= 37.5 per unit, while the L type pays only 35.

3. (a) $K^* = 15$. Note that in stage 2, there is a Cournot game where firm 1 chooses q_1 to maximize $(9 - (q_1 + q_2) - c_1)q_1$ and firm 2 chooses q_2 to maximize $(9 - (q_1 + q_2) - 6)q_2$. Maximizing these expressions, the best response curves are given by

$$q_1 = \frac{9 - c_1 - q_2}{2}$$

$$q_2 = \frac{3 - q_1}{2}$$

as long as these are non-negative. If $c_1 = 6$ then $q_1 = q_2 = 1$ so firm 1 makes profit $\pi_1 = 1$. If $c_1 = 1$ then $q_1 = 4$ and $q_2 = 0$ so firm 1 makes profit equal to π_1 =16-K. Thus, firm 1 will invest in the new technology if $16 - K \geq 1$. So the biggest K is 15. (b) Yes: because strategies are strategic substitutes and investment makes firm 1 tough, he uses a Top Dog strategy. Notice that if there was no entrant, firm 1's monopoly profit would be $\frac{9}{4}$ without investment and 16-K with investment, so he would invest if 16-K$\geq \frac{9}{4}$, i.e., if $K \leq 16 - \frac{9}{4}$. Thus, there is *overinvestment* when K is slightly smaller than K^*, specifically when $16 - \frac{9}{4} < K < 15$.

4. *Green-Porter model of collusion.* (a) In the model, a firm doesn't know the true state of demand, neither can it oberve what the other firm is doing. If a firm suffers a loss of customers, it cannot rule out the possibility that the other firm has "cheated" (made a secret price-cut). If the state of the world is such that demand is very low, there must be a price war. This cannot be avoided, because then firms could make secret price-cuts with impunity.

(b) Recover the "conduct" parameter θ as follows:

$$\theta = \hat{\alpha}_1(\exp(-\hat{\beta}_3) - 1)$$

Note that $\hat{\beta}_3 > 0$. Thus $\exp(-\hat{\beta}_3) - 1 < 0$. Due to the correlation between P_t and u_{1t}, most likely α_1 is biased upwards (less negative). Thus, the recovered θ is biased downwards (less positive). This could explain Porter's small estimate of θ.

5. (a) *Standard logit.* IIA says that the introduction of a new alternative reduces the choice probabilities of all the other alternatives by the same percentage. In the the standard logit model the choice probability is

$$\sigma_j = \frac{\exp\left(\beta' X_j - \alpha p_j\right)}{\sum_{q \in \mathcal{J}} \exp\left(\beta' X_q - \alpha p_q\right)} \tag{1}$$

If we introduce a new alternative so the new choice set becomes $\mathcal{J}'$, using (1) the relative change in choice probability is

$$\frac{\sigma_j'}{\sigma_j} = \frac{\sum_{q\in\mathcal{J}} \exp\left(\beta' X_q - \alpha p_q\right)}{\sum_{q\in\mathcal{J}'} \exp\left(\beta' X_q - \alpha p_q\right)}$$

This is the same for any j (as the right hand side does not depend on j), so IIA holds. That this is unrealistic can be seen in the red bus/blue bus example. If the original choice set is $\mathcal{J} = \{car, red\ bus\}$ and the new choice set is $\mathcal{J}' = \{car, red\ bus, blue\ bus\}$ then we expect that those who previously took the red bus will split 50-50 among blue and red bus, but the people who drove a car before will still drive a car. But this would contradict IIA.

(b) *Mixed logit.* By differentiating the expression (1) with respect to p_k, we obtain the cross-price elasticity $e_{jk} = \alpha\sigma_k p_k$. Thus, cross-price elasticities for the two types are $e^R_{jk} = \alpha^R \sigma^R_k p_k$ and $e^P_{jk} = \alpha^P \sigma^P_k p_k$. Since $\alpha^R < \alpha^P$ and $\sigma^P_k = \sigma^R_k$, $e^R_{jk} < e^P_{jk}$.